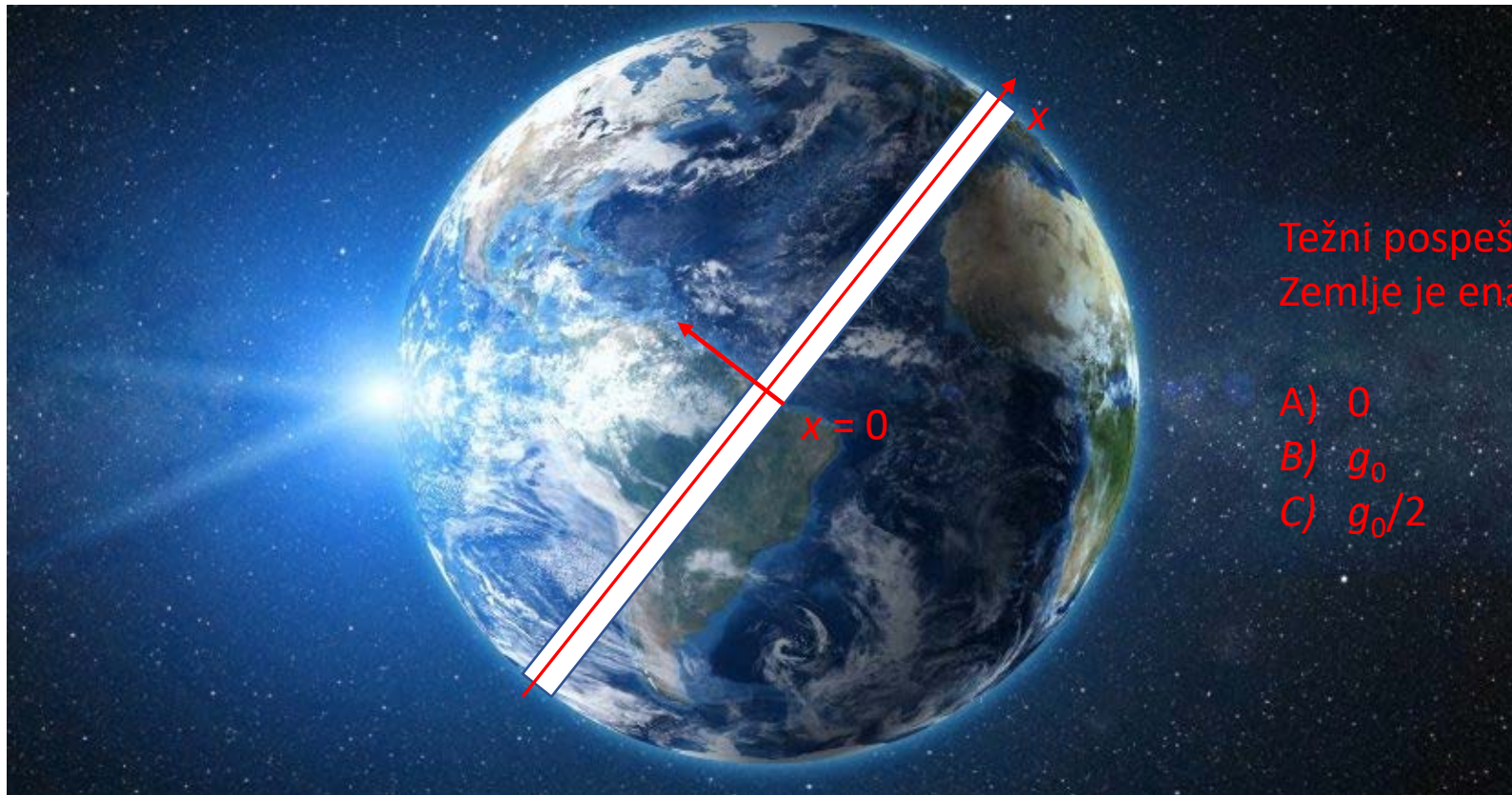


Nihanje

Zgradimo tunel skozi Zemljo – naj gre skozi središče – in izračunamo kaj se zgodi (hitrost, pospešek), ko skočimo v tak tunel. Sile upora zanemarimo, upoštevajmo le gravitacijo.

<https://www.bbc.com/future/article/20190503-the-deepest-hole-we-have-ever-dug#:~:text=This%20is%20the%20Kola%20Superdeep,of%20souls%20tortured%20in%20hell.> (12 km)

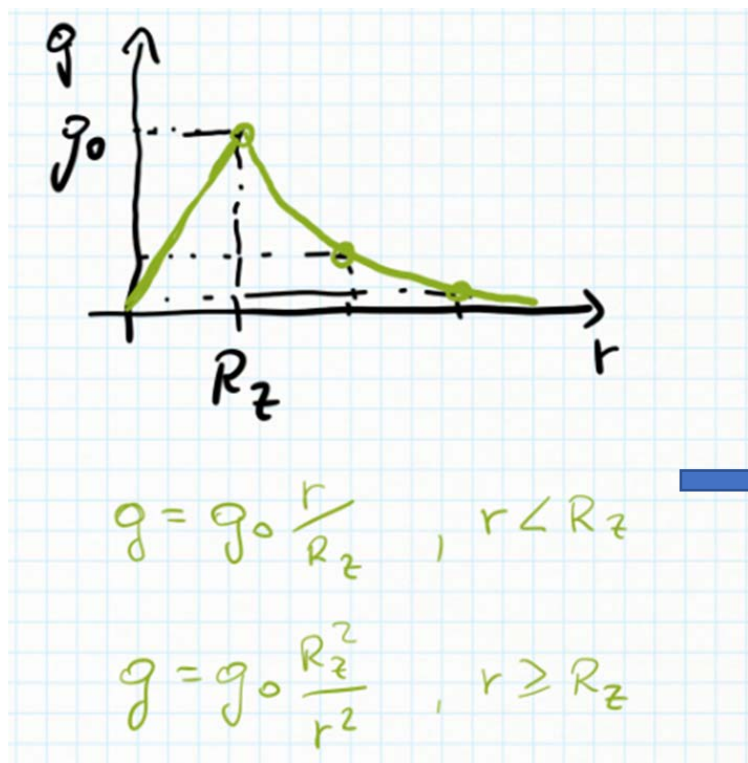


Težni pospešek v sredini Zemlje je enak:

- A) 0
- B) g_0
- C) $g_0/2$

Nihanje

Zgradimo tunel skozi Zemljo – naj gre skozi središče – in izračunamo kaj se zgodi (hitrost, pospešek), če skočimo v tak tunel. Sile upora zanemarimo, upoštevajmo le gravitacijo.



A hand-drawn diagram on grid paper showing a circular cross-section of the Earth. A horizontal red line represents a tunnel passing through the center. A mass m is shown on the right side of the tunnel. Two blue arrows represent gravitational forces: $\vec{F}_g$ pointing left from the mass towards the center, and another $\vec{F}_g$ pointing right from the center towards the left edge. A red dot marks the center, and a red label $x = \phi$ is written below it. A red arrow labeled x points to the right along the tunnel. Below the diagram, the text "Zanima nas $x(t)$!" is written in blue.

$F = -\frac{g_0}{R_z} \cdot x \cdot m$

II. NZ: $\Sigma \vec{F} = m\vec{a}$

v smeri osi x :

$$-\frac{g_0}{R_z} \cdot x \cdot m = m a$$

↑

komplikirano!

pospešek NI konstanten

Nihanje – tunel skozi Zemljo

Rešujemo

$$-\frac{g_0}{R_z} x = a = x''$$

DRUGI ODVOD
x-a po času.

$$x'' + \frac{g_0}{R_z} x = 0.$$

Kakšna funkcija $x(t)$
zadaja enačbi?

- polinom?
- eksponentna?
- sinus, kosinus!

ČE ČAS ŠTEJEMO OD TRENUTKA, KO $x = R_z$,
POTEM JE KOSINUS BOLJŠA IZBIRA:

$$x(t) = R_z \cdot \cos \omega_0 t, \quad \omega_0 = ?$$

Nihanje – tunel skozi Zemljo

Izračunamo

$$x = R_z \cos \omega_0 t$$
$$x' = v(t) = -R_z \cdot \omega_0 \sin \omega_0 t$$
$$x'' = a(t) = -R_z \omega_0^2 \cos \omega_0 t$$

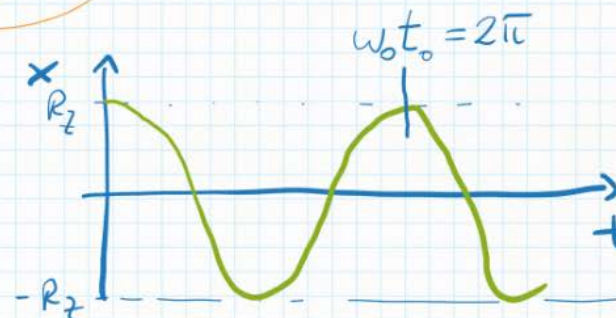
$$x'' + \frac{g_0}{R_z} x = 0$$

$$-R_z \omega_0^2 \cos \omega_0 t + \frac{g}{R_z} R_z \cos \omega_0 t = 0$$

$$\omega_0^2 = \frac{g_0}{R_z}$$

KROŽNA FREKVENCA

našljemo:



Prepoznamo nihajni čas: $t_0 = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{R_z}{g_0}} = 84 \text{ min}$

in frekvenco nihanja $\nu = \frac{1}{t_0}$

Nihanje – tunel skozi Zemljo

Pojmi, ki si jih zapomnimo:

- krožna frekvenca ω_0
- frekvenca ν_0 , $\omega_0 = 2\pi\nu_0$
- nihajni čas

in rešitev enačbe: $x'' + \omega_0^2 x = 0$

je harmonično nihanje

$$x_0 \sin \omega_0 t$$

ali

$$x_0 \cos \omega_0 t$$

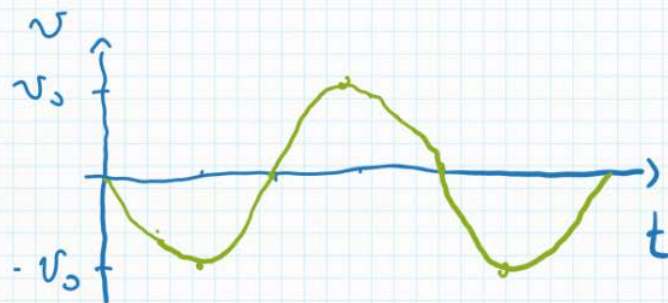
↑
amplituda nihanja.

Nihanje – tunel skozi Zemljo

Kako je to hitrost $v(t)$?

$$v(t) = x' = (R_z \cos \omega_0 t)' = -\underbrace{R_z \omega_0}_{v_0} \sin \omega_0 t$$

v_0 ... ali $v_{\max}$, amplituda nihanja hitrosti



Upoštevajmo:

$$\omega_0 = \sqrt{\frac{g_0}{R_z}}$$

in dobimo

$$v_0 = R_z \sqrt{\frac{g_0}{R_z}} =$$

$$= \sqrt{R_z \cdot g_0} = v_I = 7.9 \text{ km/s}$$

Nihanje – tunel skozi Zemljo

Preden izračunamo pospešek $a(t)$, premislimo!

Največji pospešek telesa pri padanju skozi tunel bo:

- A) g_0 (okoli 10 m/s^2)
- B) $g_0/2$
- C) veliko več od g_0

Nihanje – tunel skozi Zemljo

Ima naslednje še pospeške:

$$a(t) = v'(t) = x''(t) = (-R_0 \omega_0 \sin \omega_0 t)' = \underline{-R_0 \omega_0^2 \cos \omega_0 t} = -\omega_0^2 x(t)$$

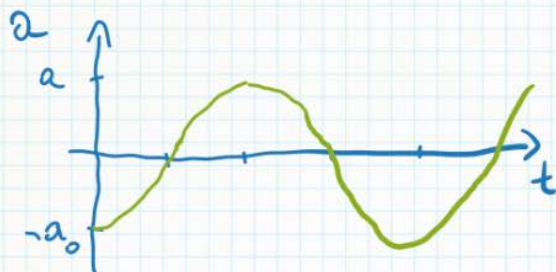
amplituda
nihanja pospeška.

upoštevajmo

$$\omega_0^2 = \frac{g_0}{R_z}$$

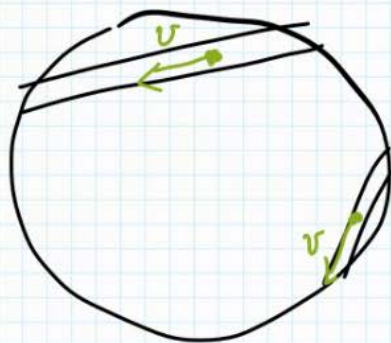
in vidimo

$$a_0 = g_0$$



Nihanje – tunel skozi Zemljo

Kaj če tunel strojamo, in ni skozi središče
Zemlje?

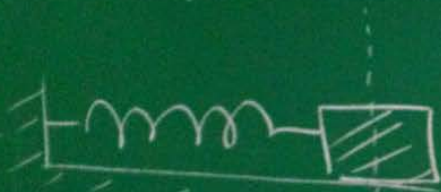


Zmagnitajo se pospeški
in hitrosti.

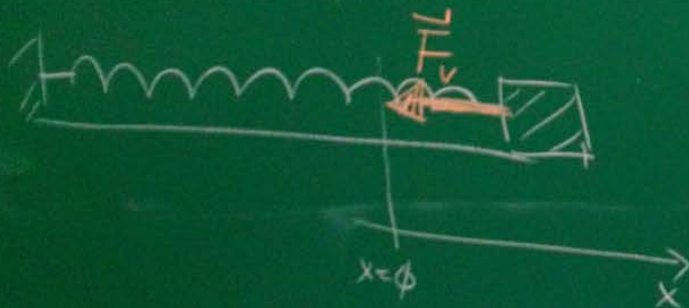
Čas potovanja ostane
enak! $t_0 = 84 \text{ min.}$

① TUNEL SKOZI ZEMLJO

② VIJAČNA VŹMET



NERAŽTEG-
NŹENA VŹMET $x = \phi$, RAVNODVESNA LEŖA



$$F_v = -k \cdot x$$

II. NZ:

$$-k x = m x'' \quad / : m$$

$$x'' + \left(\frac{k}{m} \right) x = \phi$$

$$\omega_0^2 = \frac{k}{m}, \text{ NIHAJNI \u010das:}$$

$$t_0 = 2\pi \omega_0^{-1} = 2\pi \sqrt{\frac{m}{k}}$$

③ POL

Polžasta vzmet

OLŽASTA VZMET



RAVNOVESNA
LEGA



NAVOR NA KAZALEC

(=TELO, KI NIHA) JE

$$M = -D\varphi$$

KER NAVOR HOČE ZASUKATI
NIHALO V RAVNOVESNO LEGO.

II. NZ ZA KROŽENJE

$$-D\varphi = J \cdot \underset{\substack{\uparrow \\ \text{KOTNI POSPEŠEK}}}{\alpha} = J \varphi'' \quad / : J$$

J ... vztrajnostni moment nihala

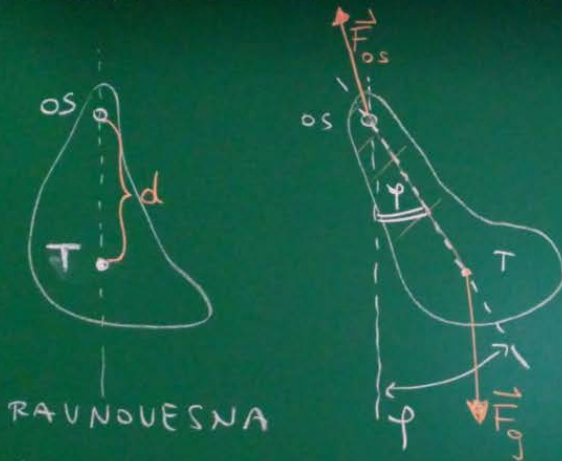
$$\varphi'' + \frac{D}{J} \varphi = 0 \quad \dots \text{iščemo } \varphi(t)$$

↳ PREPOZNAVAMO, DA JE

TO ZNOVA ENAČBA NIHANJA:

$$\varphi = \varphi_0 \sin \omega_0 t \quad ; \quad \omega_0 = \sqrt{\frac{D}{J}} \quad \text{oz} \quad t_0 = 2\pi \sqrt{\frac{J}{D}}.$$

4) TEŽNO ali FIZIČNO NIHALO



RAVNOVESNA
LEGA

d = razdalja med osjo in
težiščem.

NOVOR SILE $\vec{F}_{os}$

GLEDE NA OS = ϕ

$$M_{\vec{F}_g} = -mg \cdot d \cdot \sin \phi$$

$$-mgd \sin \phi = J \cdot \phi'' \quad \text{II. N.Z. ZA KROŽENJE}$$

$$\phi'' + \left(\frac{mgd}{J} \right) \sin \phi = 0$$

NE DOBIMO SINUSNEGA HARMONIČNEGA
NIHANJA,

RAZEN, KO JE KOT ZASUKA

MAJHEN IN VELJA $\sin \varphi \approx \varphi$ (V RADIANTIH)

$$\text{mpr. } \varphi = 10^\circ = 0,175$$

$$\sin 10^\circ = 0,174$$

$$\sin \varphi \approx \varphi + (-) \frac{\varphi^3}{6} +$$

→ ZA MAJHNE KOTE (mpr. da je $\varphi < 10^\circ$)

JE DOBER PRIBLIŽEK $\sin \varphi = \varphi$

in enačba za nihanje TEŽNEGA NIHALA

$$\varphi'' + \frac{mgd}{J} \varphi = 0$$

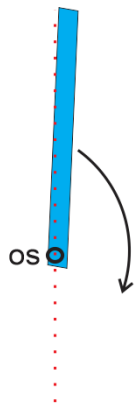
POTEM PA JE TO
HARMONIČNO NIHANJE

$$\omega_0 = \sqrt{\frac{mgd}{J}} \quad \text{oz.} \quad t_0 = 2\pi \sqrt{\frac{J}{mgd}}$$

NIČNEGA

Popravki zaradi približka $\sin\phi \approx \phi$ iz učbenika Ohanian, Physics.

Pravi nihajni časi so za toliko odstotkov večji od izračunanih po približku $\sin\phi \approx \phi$ kakor piše v tabeli:



nihajni čas je skoraj 2-krat večji od izračunanega po formuli, ki smo jo izpeljali.

Finally, we emphasize that the approximation contained in Eq. (53) is valid only for small angles. If the amplitude of oscillation is more than a few degrees, then Eq. (53) fails, and the motion of the pendulum deviates from simple harmonic motion. For instance, at large amplitudes the period of the pendulum depends on the amplitude — the larger the amplitude, the larger the period. This lack of isochronism indicates that the pendulum does not behave any more as a simple harmonic oscillator. Table 15.1 shows how the period of a pendulum increases with amplitude.

Table 15.1 INCREASE OF PERIOD OF SIMPLE PENDULUM

Amplitude	Increase of period
0°	0.0%
15°	0.4
30°	1.7
45°	4.0
60°	7.3
90°	18.0
135°	52.8
175°	187.7
180°	∞

$$t_0 = 2\pi \sqrt{\frac{J}{mgd}}$$

Primer: os



$$d = \frac{l}{2}$$

$$t_0 = 2\pi \sqrt{\frac{\frac{1}{3} m l^2}{m g l/2}} = 2\pi \sqrt{\frac{2l}{3g}} = 2\pi \sqrt{\frac{2 \cdot 1,1 \text{ m}}{3 \cdot 9,8 \text{ m/s}^2}} = 1,7 \text{ s}$$

Meritev: $10 t_0 = 17,5 \text{ s}$

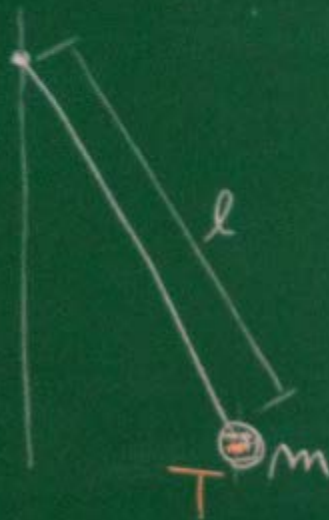
več meritev

$$10 t_0 = 16,8 \text{ s}$$

$$t_0 = 1,68 \text{ s} \approx 1,7 \text{ s}$$

potem

5) MATEMATIČNO ALI NITNO NIHALO



→ majhna kroglica

$$R \ll l$$

potem je njen $J = ml^2 + \underbrace{\frac{2}{5}mR^2}_{\text{zanemarimo.}}$

→ ma lahke vrvice

$$m_{\text{vrvica}} \ll m_{\text{kroglice}}$$

potem:
$$t_0 = 2\pi \sqrt{\frac{ml^2}{mgl}} = 2\pi \sqrt{\frac{l}{g}} \cdot$$

DELAJTO se, kakor da ne poznamo
izraza za nihajni čas

Predpostavimo potenčno odvisnost:

$$t_0 = A l^\alpha / \log$$

$$\log t_0 = \log(A l^\alpha) = \log A + \log l^\alpha$$

$$\log t_0 = \log A + \alpha \cdot \log l$$

$$y = m + b x$$

12 meritev

$$b = 0,44 \quad (\text{pričakovali bi } 0,5 \text{ zaradi } \sqrt{l})$$

$$m = \log A = 0,31$$

$$\text{ker poznamo izraz } t_0 = \frac{2\pi}{\sqrt{g}} \sqrt{l}$$

$$\log A = 0,31$$

$$A = 10^{0,31} = 2,04 = \frac{2\pi}{\sqrt{g}}$$

$$g = \left(\frac{2\pi}{2,04} \right)^2 = 9,5 \text{ m/s}^2$$

ENERGIJA PRI NIHANJU

↳ PRI NIHALIH SE SPREMINJA

KINETIČNA ENERGIJA V POTENCIALNO

(tudi PROŽNOSTNO) in OBRATNO

$$x(t) = x_0 \cos \omega_0 t$$

$$v(t) = -x_0 \omega_0 \sin \omega_0 t$$

Imejmo nihalo na vijačno vzmet

Seštejmo $W_k + W_{pr} = W_{celotna}$

$$W_{celotna} = \frac{m v(t)^2}{2} + \frac{k x(t)^2}{2} = \frac{m \cdot x_0^2 \omega_0^2 \sin^2 \omega_0 t}{2} + \frac{k \cdot x_0^2 \cos^2 \omega_0 t}{2} =$$

upoštevajmo $\omega_0^2 = \frac{k}{m}$

$$= \frac{x_0^2}{2} \left(\cancel{m} \cdot \frac{\cancel{h}}{\cancel{m}} \cdot \sin^2 \omega_0 t + h \cos^2 \omega_0 t \right) = \frac{x_0^2 \cdot h}{2} \underbrace{(\sin^2 \omega_0 t + \cos^2 \omega_0 t)}_{\text{vedno enako 1}}$$

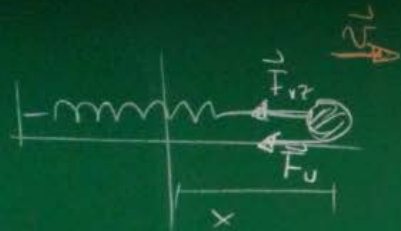
$$W_{pr(max)} = W_{k(max)}$$

||
maximalna
prožnostna energija

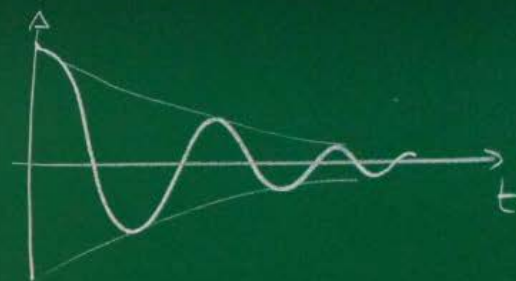
ČE JE POTENCIALNA ENERGIJA $\propto$ KVADRATU ODPIKA,
DOBIMO SINUSNO/KOSINUSNO HARMONIČNO NIHANJE

DUŠENO NIHANJE

imamo neko zaviralno silo, mpr. če jo lahko izkažimo $\vec{F}_{upor} = -\gamma \vec{v}$



$$-kx - \underset{\text{HITROSŤ}}{\gamma x'} = m \underset{\text{POSPEŠEK}}{x''} \quad / : m$$



$$x'' + \underbrace{\frac{2\gamma}{2m}}_{\omega_0^2} x' + \underbrace{\frac{k}{m}}_{\omega_0^2} x = 0$$

β = koeficient dušenia

rešenie je:

$$x(t) = x_0 e^{-\beta t} \cos \omega t$$

$$\omega^2 = \omega_0^2 - \beta^2$$

LASTNA FREKV. ČE BI BIL NEDUŠENO.

VSILJENO NIHANJE

Pomembno je v katerih trenutkih in s kakšno frekvenco nihalu dodajamo energijo.

ma, bo sila s katero vsiljujemo oblike

$$F = F_0 \sin \omega t$$

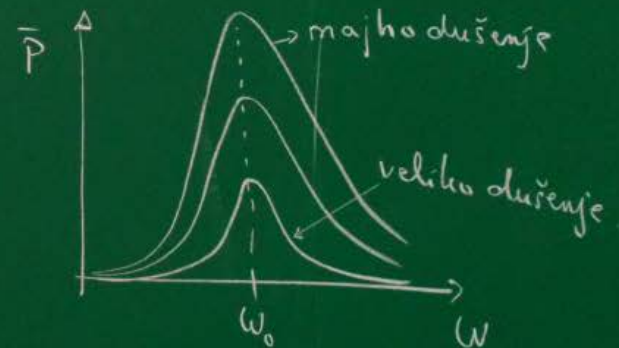
Dobimo diferencialno enačbo:

$$x'' + 2\beta x' + \omega_0 x = \frac{F_0}{m} \sin \omega t$$

Povprečna moč, ki jo vsiljujemo nihalu:

$$\bar{P} = K_{\text{const}} \frac{\omega^2}{(\omega_0^2 - \omega^2)^2 + 4\beta^2 \omega^2}$$

Dobimo resonančno krivuljo



a) ko $\omega < \omega_0 \Rightarrow$ nihalo miha v fazi (sočasno)
skatero $\downarrow$ Lastna
vsiljnjemo $\downarrow$ z vsiljeno silo

b) v resonanci
 $\omega = \omega_0 \Rightarrow$ nihalo zaostaja za 90°

c) ko $\omega > \omega_0 \Rightarrow$ nihalo zaostaja za 180°